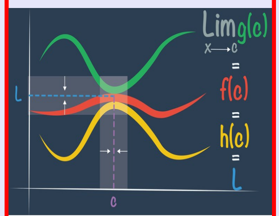
Calculus I

Lecture 12



Feb 19-8:47 AM

Class Quiz 11 $\boxed{\text{Open Notes}}$

Find equation of the tangent line to the graph of $f(x) = x^4 + 2x^2 - x$ at $x=1$ in Slope-Int. Form.

$f(1) = 1^4 + 2(1)^2 - 1 = 1 + 2 - 1 = \boxed{2}$

$f'(x) = 4x^3 + 2\cdot 2x - 1$

$f'(x) = 4x^3 + 4x - 1$

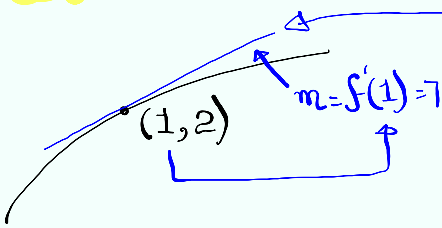
$f'(1) = 4(1)^3 + 4(1) - 1 = \boxed{7}$

$y - y_1 = m(x - x_1)$

$y - 2 = 7(x - 1)$

$y - 2 = 7x - 7$

$\boxed{y = 7x - 5}$



Mar 19-11:00 AM

Class Quiz 12

Given $f(x) = x + 2\cos x$

1) Find $f'(x) = 1 + 2 \cdot -\sin x = \boxed{1 - 2\sin x}$

2) Solve $f'(x) = 0$ on $[0, 2\pi]$.

$1 - 2\sin x = 0$

$-2\sin x = -1$

$\sin x = \frac{-1}{-2}$

$\sin x = \frac{1}{2}$ QI & QII

Ref. Angle $\frac{\pi}{6}$

QI $x = \boxed{\frac{\pi}{6}}$

QII $x = \pi - \frac{\pi}{6} = \boxed{\frac{5\pi}{6}}$

Mar 24-7:47 AM

Differentiation Rules:

1) $\frac{d}{dx}[c] = 0$

2) $\frac{d}{dx}[x] = 1$

3) $\frac{d}{dx}[x^n] = n x^{n-1}$

4) $\frac{d}{dx}[c f(x)] = c \frac{d}{dx}[f(x)]$

5) $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$

6) $\frac{d}{dx}[f(x) \cdot g(x)] = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

Product Rule

$\frac{d}{dx}[x^2 \sin x] = 2x \cdot \sin x + x^2 \cdot \cos x$

$\frac{d}{dx}[(x^2 - 5x)(x^3 + 8)] = (2x - 5)(x^3 + 8) + (x^2 - 5x) \cdot 3x^2$

$= 2x^4 + 16x - 5x^3 - 40 + 3x^4 - 15x^3$

$= \boxed{5x^4 - 20x^3 + 16x - 40}$

Mar 24-9:10 AM

Find $\frac{dy}{dx} = y'$

1) $y = 18$ $\qquad \boxed{y' = 0}$

2) $y = x\sqrt{x}$ $\qquad y = x\,x^{\frac{1}{2}}$ $\qquad y = x^{\frac{3}{2}}$ $\qquad y' = \frac{3}{2}x^{\frac{3}{2}-1}$

$y' = \frac{3}{2}x^{\frac{1}{2}}$

$y' = \frac{3}{2}\sqrt{x}$

3) $y = \sin x \cdot \cos x$

$y' = \cos x \cdot \cos x + \sin x \cdot (-\sin x)$

$= \cos^2 x - \sin^2 x \rightarrow y' = \cos 2x$

Mar 24-9:18 AM

Find $f'(x)$ & $f''(x)$

First Derivative $\qquad$ Second Derivative

1) $f(x) = x^2 - 4x$ $\qquad f'(x) = \boxed{2x - 4}$

$f''(x) = 2 - 0 = \boxed{2}$

2) $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 5$ $\qquad f'(x) = \frac{1}{3}\cdot 3x^2 + \frac{1}{2}\cdot 2x - 0$

$\boxed{f'(x) = x^2 + x}$

$\boxed{f''(x) = 2x + 1}$

3) $f(x) = (\overset{A\ +B}{x^2+5})(\overset{A\ -B}{x^2-5})$

$f'(x) = 2x(x^2-5) + (x^2+5)\cdot 2x$

$= 2x[x^2 \cancel{-5} + x^2 \cancel{+5}]$

$= 2x \cdot 2x^2 = \boxed{4x^3}$ ✓

$f''(x) = 4\cdot 3x^{3-1}$

$= \boxed{12x^2}$ ✓

$\rightarrow f(x) = (x^2)^2 - (5)^2$

$f(x) = x^4 - 25$

$f'(x) = 4x^3 - 0$

$= 4x^3$

$f''(x) = 4\cdot 3x^2$

$= 12x^2$

Mar 24-9:24 AM

Differentiation Rules:

1) $\frac{d}{dx}[c] = 0$

2) $\frac{d}{dx}[x] = 1$

3) $\frac{d}{dx}[x^n] = n x^{n-1}$

4) $\frac{d}{dx}[c f(x)] = c \frac{d}{dx}[f(x)]$

5) $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$

6) $\frac{d}{dx}[f(x) \cdot g(x)] = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

Product Rule

7) $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{[g(x)]^2}$

Quotient Rule

$$\frac{d}{dx}\left[\frac{2x+3}{3x-1}\right] = \frac{2(3x-1) - (2x+3) \cdot 3}{(3x-1)^2} = \frac{\cancel{6x}-2-\cancel{6x}-9}{(3x-1)^2} = \boxed{\frac{-11}{(3x-1)^2}}$$

Mar 24-9:10 AM

$$f(x) = \frac{x^2 - 4x}{x^2 + 3x + 2}$$

Find $f'(x) = \frac{(2x-4)(x^2+3x+2) - (x^2-4x)(2x+3)}{(x^2+3x+2)^2}$

$$= \frac{\cancel{2x^3} \cancel{+6x^2} \cancel{+4x} \cancel{-4x^2} \cancel{-12x} - 8 \cancel{-2x^3} \cancel{-3x^2} \cancel{+8x^2} \cancel{+12x}}{(x^2+3x+2)^2}$$

$$= \frac{7x^2 + 4x - 8}{(x^2+3x+2)^2}$$

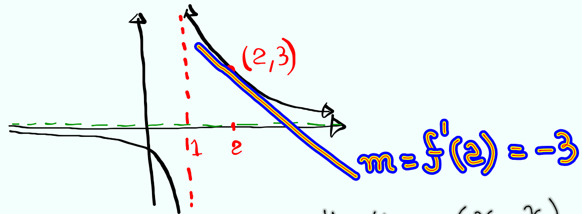
Mar 24-9:42 AM

$f(x) = \dfrac{3}{x-1}$

1) Find $f(2) = \dfrac{3}{2-1} = \dfrac{3}{1} = 3$

2) Find $f'(x) = \dfrac{\underbrace{0(x-1)}_{\to 0} - 3 \cdot 1}{(x-1)^2} = \boxed{\dfrac{-3}{(x-1)^2}}$

3) Find $f'(2) = \dfrac{-3}{(2-1)^2} = \dfrac{-3}{1} = \boxed{-3}$



$y - y_1 = m(x - x_1)$

$y - 3 = -3(x-2)$

$\boxed{y = -3x + 9}$

Mar 24-9:49 AM

$f(x) = \dfrac{x}{x+2}$

1) Find $f(2) = \dfrac{2}{2+2} = \dfrac{2}{4} = \dfrac{1}{2}$

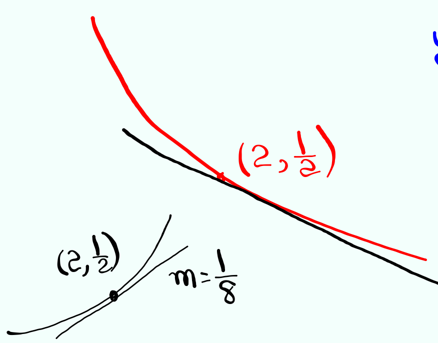
2) Find $f'(2)$ $\qquad f'(x) = \dfrac{1(x+2) - x \cdot 1}{(x+2)^2}$

$f'(2) = \dfrac{2}{(2+2)^2} = \dfrac{2}{16} = \boxed{\dfrac{1}{8}} \qquad = \dfrac{x+2-x}{(x+2)^2} = \dfrac{2}{(x+2)^2}$

$y - \frac{1}{2} = \frac{1}{8}(x-2)$

$y = \frac{1}{8}x - \frac{2}{8} + \frac{1}{2}$

$y = \frac{1}{8}x + \frac{2}{8}$



$m = f'(2) = \frac{1}{8}$

$\boxed{y = \frac{1}{8}x + \frac{1}{4}}$

Mar 24-9:58 AM

$$f(x) = \frac{\sin x}{1 + \cos x}$$

1) Find $f\left(\frac{\pi}{2}\right)$: $\dfrac{\sin\frac{\pi}{2}}{1 + \cos\frac{\pi}{2}} = \dfrac{1}{1+0} = \dfrac{1}{1} = \boxed{1}$

$\frac{\pi}{2} \rightarrow 90^\circ$

2) Find $f'\left(\frac{\pi}{2}\right)$

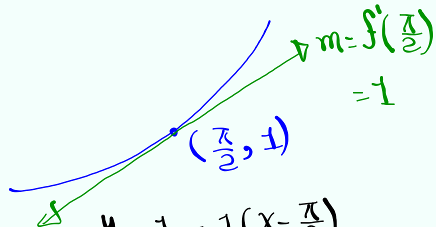
$$f'(x) = \frac{\cos x(1 + \cos x) - \sin x(-\sin x)}{(1 + \cos x)^2}$$

$$= \frac{\cos x + \cos^2 x + \sin^2 x}{(1 + \cos x)^2}$$

$$= \frac{\cos x + 1}{(1 + \cos x)^2}$$

$$= \boxed{\frac{1}{1 + \cos x}}$$

$$= \frac{1}{1 + \cos\frac{\pi}{2}} = \frac{1}{1+0} = \boxed{1}$$



$$y - 1 = 1\left(x - \frac{\pi}{2}\right)$$

$$\boxed{y = x - \frac{\pi}{2} + 1}$$

Mar 24-10:07 AM

Find a formula for $\frac{d}{dx}[\tan x]$

$$\frac{d}{dx}[\sin x] = \cos x$$

$$\frac{d}{dx}[\cos x] = -\sin x$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\frac{d}{dx}[\tan x] = \sec^2 x$$

$$\frac{d}{dx}[\tan x] = \frac{d}{dx}\left[\frac{\sin x}{\cos x}\right]$$

$$= \frac{\cos x \cdot \cos x - \sin x \cdot (-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} = \left[\frac{1}{\cos x}\right]^2 = \sec^2 x$$

Mar 24-10:17 AM

find $\frac{d}{dx}[\sec x]$

$$\frac{d}{dx}[\sec x] = \frac{d}{dx}\left[\frac{1}{\cos x}\right]$$

$$= \frac{\cancel{0 \cdot \cos x}^{\,0} - 1 \cdot (-\sin x)}{\cos^2 x}$$

$$= \frac{\sin x}{\cos^2 x} = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$$

$$= \sec x \cdot \tan x$$

$\frac{d}{dx}[\sec x] = \sec x \tan x$

Mar 24-10:23 AM

$\frac{d}{dx}[\sin x] = \cos x$

$\frac{d}{dx}[\cos x] = -\sin x$

$\frac{d}{dx}[\tan x] = \sec^2 x$

$\frac{d}{dx}[\cot x] = -\csc^2 x$

$\frac{d}{dx}[\sec x] = \sec x \tan x$

$\frac{d}{dx}[\csc x] = -\csc x \cot x$

Mar 24-10:28 AM

$f(x) = (2x-3)^2$

1) find $f(2) = (2\cdot 2-3)^2 = (4-3)^2 = 1^2 = \boxed{1}$

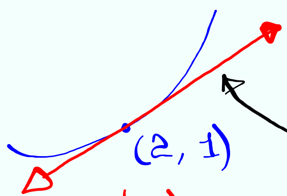
2) find $f'(x)$

$f(x) = (2x-3)(2x-3)$

$f'(x) = 2(2x-3) + (2x-3)\cdot 2$

$f'(x) = 4(2x-3)$

$= \boxed{8x-12}$



$m = f'(2) = 8(2)-12 = \boxed{4}$

$y-1 = 4(x-2)$

$\boxed{y = 4x-7}$

Mar 24-10:32 AM

Find the eqn of the tan. line to the graph of $f(x) = \sec x$ at $x = \frac{\pi}{4}$.

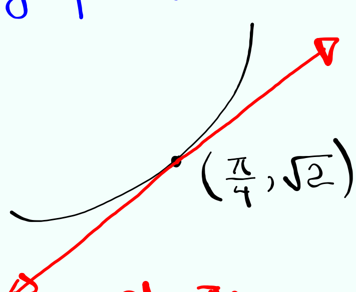
$f(\frac{\pi}{4}) = \sec\frac{\pi}{4}$

$= \frac{1}{\cos\frac{\pi}{4}}$

$= \frac{1}{\frac{\sqrt{2}}{2}}$

$= \frac{2}{\cancel{\sqrt{2}}}\cdot\frac{\sqrt{2}}{\cancel{\sqrt{2}}}$

$= \sqrt{2}$



$f(x) = \sec x$

$f'(x) = \sec x \tan x$

$m = f'(\frac{\pi}{4})$

$= \sec\frac{\pi}{4}\tan\frac{\pi}{4} = \sqrt{2}\cdot 1 = \sqrt{2}$

$y - y_1 = m(x - x_1)$

$y - \sqrt{2} = \sqrt{2}(x - \frac{\pi}{4}) \rightarrow \boxed{y = \sqrt{2}(x - \frac{\pi}{4}) + \sqrt{2}}$

Mar 24-10:37 AM

$f(x) = \dfrac{x+4}{2x-4}$

1) $f(0) = \dfrac{0+4}{2(0)-4} = \dfrac{4}{-4} = \boxed{-1}$

2) $\lim_{x \to \infty} f(x) = \dfrac{\infty}{\infty}$ I.F.

$\lim_{x\to\infty} \dfrac{\frac{x}{x}+\frac{4}{x}}{\frac{2x}{x}-\frac{4}{x}} = \lim_{x\to\infty} \dfrac{1+\frac{4}{x}\to 0}{2-\frac{4}{x}\to 0} = \boxed{\dfrac{1}{2}}$

3) $f'(x) = \dfrac{1(2x-4)-(x+4)\cdot 2}{(2x-4)^2}$

$= \dfrac{\cancel{2x}-4-\cancel{2x}-8}{(2x-4)^2} = \dfrac{-12}{(2x-4)^2} = \dfrac{-12}{[2(x-2)]^2}$

$= \dfrac{-\cancel{12}\,3}{\cancel{4}(x-2)^2}$

$= \boxed{\dfrac{-3}{(x-2)^2}}$

Mar 24-10:45 AM

find $f'(x)$

1) $f(x) = \pi$

$f'(x) = 0$

2) $f(x) = \pi R^2$

$f'(x) = 0$

3) $f(x) = t^3 - 5t^2 + 8t$

$f'(x) = 0$

4) $f(x) = \overbrace{\sec x \cos x}$

$f(x) = 1$

$f'(x) = 0$

Mar 24-10:52 AM

find y', y'', and y''' for $y = \sin x + \cos x$.

$y = \sin x + \cos x$

$y' = \cos x - \sin x$

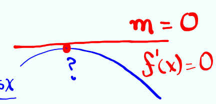
$y'' = -\sin x - \cos x$

$y''' = -\cos x - (-\sin x) = -\cos x + \sin x$

Mar 24-11:00 AM

(x, y) $[0, 2\pi]$

Find $\boxed{\text{points}}$ on the graph of $f(x) = \dfrac{\cos x}{2 + \sin x}$ at which the tangent line is horizontal.



$$f'(x) = \frac{-\sin x(2+\sin x) - \cos x \cdot \cos x}{(2+\sin x)^2}$$

$$= \frac{-2\sin x - \sin^2 x - \cos^2 x}{(2+\sin x)^2} = \frac{-2\sin x - (\cancel{\sin^2 x + \cos^2 x})^{1}}{(2+\sin x)^2}$$

$$= \frac{-2\sin x - 1}{(2+\sin x)^2}$$

we want

$f'(x) = 0$

$-2\sin x - 1 = 0$

$\sin x = \frac{-1}{2}$

Q III & Q IV

Ref. Angle $\frac{\pi}{6}$

Q III $x = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

Q IV $x = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

Points

$\left(\frac{7\pi}{6}, ?\right)$ $f\left(\frac{7\pi}{6}\right) = \dfrac{\cos\frac{7\pi}{6}}{2 + \sin\frac{7\pi}{6}}$

$\left(\frac{11\pi}{6}, ?\right)$ $f\left(\frac{11\pi}{6}\right) = \dfrac{\cos 11\pi/6}{2 + \sin 11\pi/6}$

Google Chain Rule

Mar 24-11:04 AM